On the effect of feedback delay on limited-rate beamforming systems

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***Abstract*—The use of beamforming to enable higher data rates in telecommunications is widely appreciated, but performance gains are typically calculated assuming delay-free feedback from the receiver and neglecting processing time. This paper introduces a mathematical framework based on outage probability that measures the extent to which current channel state information is accurate. Performance gains from beamforming can then be evaluated as a function of the *currency* of system state. Results are provided for Multiple Input Single Output (MISO) and for Multiuser Multiple Input Multiple Output (MU-MIMO) systems. Outage probabilities and effective diversity orders are calculated for widely used methods of beamforming such as Transmit Antenna Selection as a function of the speed of channel variation.**

I. Introduction

Beamforming has been shown to have the potential to significantly improve throughput of wireless systems by providing transmitter and receiver with a higher quality communications channel. There have been several papers addressing the effect of quantization on channel adaptation (for example, [1], [2], [3], [4]) but very few addressing the effect of feedback delay, even though this last issue is critical to system design.

In this paper we study the average outage probability for different antennae configurations and codebook designs. We argue that the estimation of outage probability is more appropriate for real-world scenarios. Many communication systems are incapable of fine-grained adjustment of the transmission rate, therefore the comparison to a fixed transmission rate is valid. BER estimation is tainted at the upper systems layers by forward-error correction and other means of bit recovery, hence the theoretical estimation usually does not reflect user perception. The outage probability can instead make a feasible tradeoff between provided transmission rate and transmission reliability.

First, we consider multiple-input and single-output (MISO) systems with different beamforming techniques. We start by considering Random Vector Quantization (RVQ) codebooks and by deriving an analytical expression for the outage probability of RVQ beamforming. This makes it possible to analyze the tradeoff between currency (feedback delay) and codebook size for different channel types. We also consider Transmit Antenna Selection (TAS) which is simple and widely used. To contrast the above findings we use outage probability estimations for perfect beamforming (PBF) with feedback delay explored by Annapureddy *et al.* in [5]. In the following, We extend the results on perfect beamforming, RVQ beamforming and TAS to multiuser multiple input multiple output (MU-MIMO) systems.

The rest of this paper is organized as follows. Section II presents the system and channel models. In Section III we study MISO systems with delayed feedback and provide an analysis on the performance tradeoffs between beamforming schemes. The findings from Section III are used as a foundation to the study of the MU-MIMO scenarios described in Section IV. We conclude this paper in Section V.

II. System Overview

In a wireless communication system with N_t transmit antennas and N_r receive antennas, the received signal $\mathbf{r}$ is given by

$$\mathbf{r} = \tilde{\mathbf{h}}\mathbf{x}^{\dagger} + \mathbf{n} \tag{1}$$

where $\tilde{\mathbf{h}} \in \mathbb{C}^{N_t \times N_r}$ is the channel vector with the entry $\tilde{h}_{i,j}$ representing the complex channel gain between the i^{th} transmit antenna and the j^{th} receive antenna, $\mathbf{x}$ represents the transmitted signal vector, and $\mathbf{n}$ is the corresponding additive noise vector with each component following the complex Gaussian distribution, $CN(0,1)$. We assume a block fading narrowband channel. In order to introduce feedback delay, we consider the following channel model [6]:

$$\tilde{\mathbf{h}} = \rho\mathbf{h} + \sqrt{1-\rho^2}\mathbf{e} \tag{2}$$

where $\mathbf{h}$ is the channel vector at time of last estimation, $\mathbf{e}$ is the aging-related deviation, $\mathbf{h}$ and $\mathbf{e}$ are independent and each of their components follow the standard complex Gaussian distribution, $CN(0,1)$. The temporal persistence of the channel is measured by ρ and it is proportional to the time slot size between two consecutive estimations. This Gauss Markov channel model is especially useful when ρ is relatively small.

III. Outage Analysis for MISO systems with Feedback Delay

We first consider a MISO system with transmit beamforming. The system is composed of N_t transmit antennas and one receiver antenna. Bandwidth occupation is small in comparison to the coherence bandwidth. Let $\mathcal{C} = \{\mathbf{p}_1, \cdots, \mathbf{p}_N\}$ be an arbitrary (rank-one) beamforming codebook with $\mathbf{p}_i \in \mathbb{C}^{N_t}$ and a uniform energy budget $\|\mathbf{p}_i\|^2 = 1$. Thus the received signal for the beamforming system is written as

$$r = \langle \tilde{\mathbf{h}}, \hat{\mathbf{p}} \rangle x + n \tag{3}$$

where $\langle \cdot, \cdot \rangle$ is the inner product and $\hat{\mathbf{p}}$ is the selected beamformer. In this section we estimate the average outage probability for different codebook schemes . In the following, we refer to the average outage probability as P_{out}^{C} where C is the codebook type referenced.

Outage is encountered when the mutual information between sent and received symbols is lower than the system transmission rate. For the MISO system with the channel model in (3), the general instantaneous probability of system outage is given by

$$\begin{aligned} P_{\text{out}}(R,\epsilon) &= \Pr\left[\mathcal{I}(\mathbf{r},\mathbf{x}|\tilde{\mathbf{h}}) < R\right] \\ &= \Pr\left[\log_2(1+\frac{\epsilon}{N_t}|\langle \tilde{\mathbf{h}}, \mathbf{p}_i\rangle|^2) < R\right] \\ &= \Pr\left[\left|\langle \tilde{\mathbf{h}}, \mathbf{p}_i\rangle\right|^2 < \gamma_0\right] \end{aligned} \quad (4)$$

where R is the transmission rate, ϵ is the signal to noise ratio (SNR) and $\gamma_0 = \frac{2^R-1}{\epsilon/N_t}$. To incorporate feedback delay modeled in equation (2), we further write

$$\begin{aligned} P_{\text{out}}(R,\epsilon) &= \Pr\left[\left|\rho\langle \mathbf{h}, \hat{\mathbf{p}}\rangle + \sqrt{1-\rho^2}\langle \mathbf{e}, \hat{\mathbf{p}}\rangle\right|^2 < \gamma_0\right] \\ &= \Pr\left[\left|\sqrt{2\mu}\langle \mathbf{h}, \hat{\mathbf{p}}\rangle + \sqrt{2}\langle \mathbf{e}, \hat{\mathbf{p}}\rangle\right|^2 < \frac{2\gamma_0}{1-\rho^2}\right] \\ &= \Pr\left[\left|\sqrt{2\mu\gamma} + \sqrt{2}z\right|^2 < \frac{2\gamma_0}{1-\rho^2}\right] \end{aligned} \quad (5)$$

where $\mu = \frac{\rho^2}{1-\rho^2}$, $\gamma = |\langle \mathbf{h}, \hat{\mathbf{p}}\rangle|^2$ and $z = \langle \mathbf{e}, \hat{\mathbf{p}}\rangle e^{-i\angle\langle \mathbf{h}, \hat{\mathbf{p}}\rangle}$ is complex Gaussian distributed with zero mean and unit variance. So $\left|\sqrt{2\mu\gamma} + \sqrt{2}z\right|^2$ is non-central chi-square distributed with two degrees of freedom and parameter $\sqrt{2\mu\gamma}$.

Equation (5) provides us with convenient means to study the average outage probability for different codebook schemes. The original selection problem for codes remains unchanged due to the duality in selecting the code that maximizes γ, but the left-hand side of equation (5) lends itself to analytical methods as the average outage probability is tractable through the distribution of the instantaneous outage probability.

A. MISO-PBF scenario

In [5], Annapureddy *et al.* derive the outage probability for a beamforming system that is subject to feedback delay. They show that in the low SNR regime, power allocation in the direction of the CSI performs better, while uniform spatial power allocation (USPA) is beneficial in high SNR scenarios. The minimum outage probability for unconstrainted codebook sizes is

$$\begin{aligned} & P_{\text{out}}^{\text{PBF}}(R,\epsilon,\rho) \\ &= \frac{1}{(1+\mu)^{N_t-1}} \sum_{k=0}^{N_t-1} \binom{N_t-1}{k} \frac{\mu^k}{k-1} \Gamma_{k+1}(\gamma_0) \end{aligned} \quad (6)$$

where $\Gamma_k(x)$ is the lower incomplete gamma function.

B. MISO-RVQ scenario

The codebooks of RVQ systems are characterised by a fixed cardinality N and contain isotropically distributed codes. We define the tradeoff factor $\nu = \max_{\mathbf{p}\in\mathcal{C}} \frac{|\langle \mathbf{h},\mathbf{p}\rangle|^2}{\|\mathbf{h}\|^2}$ as the loss of adaptability in comparison to the perfect mutual-information maximizing code due to the limited size of the RVQ codebook. Complex Gaussian distribution of the channel vector and isotropy of the codebook yields the probability density function as [7]

$$f_\nu = N(N_t-1)\left(1-(1-\nu)^{N_t-1}\right)^{N-1}(1-\nu)^{N_t-2}. \quad (7)$$

We now extend (5) with the tradeoff factor and average over the probability density function, resulting in

$$P_{\text{out}}^{\text{RVQ}}(R,\epsilon,\rho) \quad = \sum_{k=0}^{N_t-1} A(k) \binom{N_t-1}{k} \frac{\mu^k}{k-1}\Gamma_{k+1}(\gamma_0)$$

with

$$A(k) = \int_0^1 \frac{1}{(1+\mu\nu)^{N_t-1}} \nu^k f_\nu d\nu. \quad (8)$$

Proof: Given ν, the outage probability is

$$\begin{aligned} & \Pr(\text{outage}|\,\nu) \\ &= \Pr\left[\left|\sqrt{2\mu\nu\gamma} + \sqrt{2}z\right|^2 < \frac{2\gamma_0}{1-\rho^2}\right] \\ &= \frac{1}{(1+\nu\mu)^{N_t-1}} \sum_{k=0}^{N_t-1} \binom{N_t-1}{k} \frac{(\nu\mu)^k}{k-1}\Gamma_{k+1}\left(\frac{e^R-1}{\epsilon/N_t}\right). \end{aligned} \quad (9)$$

Then, the overall outage probability for random vector quantization beamforming systems of N random vectors is

$$P_{\text{out}}^{\text{RVQ}} = \int \Pr(\text{outage}|\,\nu) f_\nu d\nu.$$

□

The diversity order is defined as

$$D = -\lim_{\epsilon\to\infty} \frac{\log P_{\text{out}}(\epsilon)}{\log \epsilon}. \quad (10)$$

We can derive the diversity order for RVQ beamforming [8] as

$$D^{\text{RVQ}} = \begin{cases} 1, & 0 \le \rho < 1, \\ N_t, & \rho = 1. \end{cases} \quad (11)$$

Remark: The diversity derived here is for the uncoded system.

Fig. 1 illustrates the significant performance decay of RVQ beamforming when channel estimation becomes inaccurate due to the delayed feedback. This figure depicts a 4×1 MISO system that applies a beamformer codebook of size eight at a transmission rate of 2 bits/s/Hz. For comparison we plot the outage probabilities for a similar 2×1 system. Note that the loss of persistence results in an equalization of the slope of the two systems, indicating the loss in diversity order of the MISO setup.

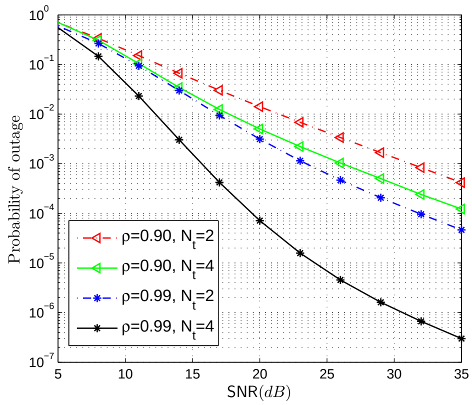

Fig. 1. Outage Probability for 4×1 and 2×1 systems with a RVQ beamforming codebook of size eight and transmission rate at 2 bits/s/Hz.

In Fig. 2 we take an opposite approach and study the feasibility of RVQ beamforming to mitigate the delayed feedback through an increase of the codebook size. The cardinality grows exponentially with lower channel persistence at a rate depending on the minimum achievable outage probability. We propose to study adaptive methods that make use of this finding in future work to allow for dynamic codebooks and amount of feedback information.

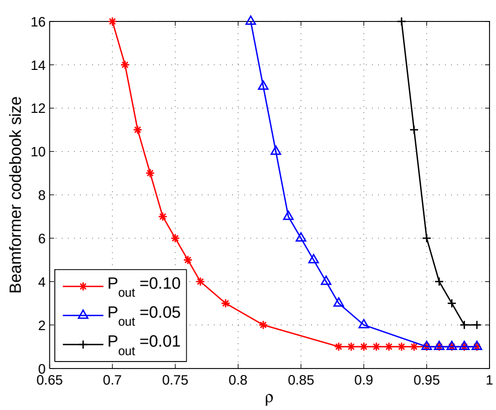

Fig. 2. Minimum RVQ codebook size for given outage probability depending on channel persistence and transmission rate at 2 bits/s/Hz.

C. MISO-TAS scenario

In pure TAS scenarios, the transmitter-side antenna is selected for which the channel gain is maximal, hence

$$\gamma_{\text{TAS}} = \max_i |h_i|^2$$

with unity power. γ_{TAS} is the largest member of a set of exponentially distributed channel gain values h_i with probability density function

$$f_{\gamma_{\text{TAS}}}(x) = N_t \left(1 - e^{-x}\right)^{N_t - 1} e^{-x}. \tag{12}$$

Based on the distribution of γ_{TAS}, we now derive the outage probability for TAS scenarios.

Proposition 1 (MISO-TAS outage probability)**.** *Consider a $N_t \times 1$ wireless fading channel employing transmit antenna selection and transmitting the signals at date rate of R bits/s/Hz with* SNR $= \epsilon$*, the outage probability in the presence of feedback delay is*

$$P_{out}^{TAS}(R, \epsilon, \rho) = N_t \sum_{k=0}^{N_t - 1} \binom{N_t - 1}{k} \frac{(-1)^k}{k+1} \left(1 - e^{-\frac{k+1}{k+1+u}\frac{2\gamma_0}{1-\rho^2}}\right). \tag{13}$$

Proof: The proof is provided in [8].□

In the presence of feedback delay ($0 < \rho < 1$), the diversity order for TAS is

$$\lim_{\epsilon \to \infty} - \frac{\log \left(\gamma_0 N_t \sum_{k=0}^{N_t - 1} \binom{N_t - 1}{k} \frac{2(-1)^k}{(k+1+u)(1-\rho^2)} \right)}{\log \epsilon} = 1. \tag{14}$$

Hence, the diversity order for TAS is given by

$$D^{\text{TAS}} = \begin{cases} 1, & 0 \le \rho < 1, \\ N_t, & \rho = 1. \end{cases} \tag{15}$$

Fig. 3 illustrates the outage probability for TAS beamforming with different channel persistencies. We encounter a similar pattern to that in Fig. 1: Performance deteriorates fast with the loss of channel persistence. At the same time, the slopes of the graphs indicate a loss in diversity order.

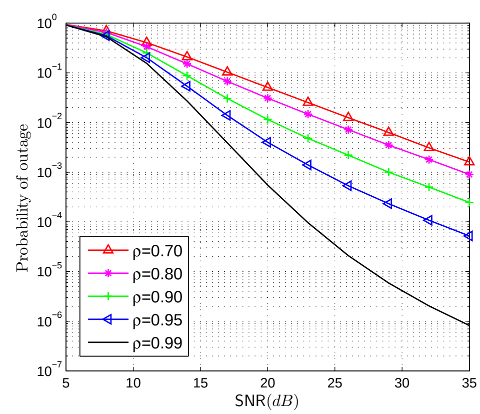

Fig. 3. Outage Probability for 4×1 systems with transmit antenna selection and transmission rate at 2 bits/s/Hz.

D. Comparison

Outage probability in a low-persistence channel ($\rho = 0.8$) is shown in Fig. 4 for perfect beamforming, RVQ beamforming and transmit antenna selection for a 4×1 MISO system. Perfect beamforming always outperforms other schemes as expected. If the codebook size is significantly large, the performance of RVQ beamforming approaches that of perfect beamforming. All schemes evaluated suffer from a loss of diversity order.

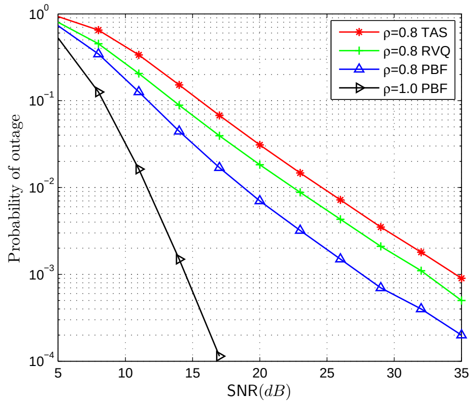

Fig. 4. Outage Probability for 4×1 systems with different beamforming schemes; transmission rate is at 2 bits/s/Hz and the RVQ beamforming codebook has eight random beamforming vectors.

IV. Outage analysis for multiuser MIMO systems with Feedback Delay

Now we consider a N_u-user system with a base station employing N_t transmit antennas and each user equipped with N_r receive antennas. We first derive the outage probabilities for multiuser MIMO systems with transmit antenna selection. Then, using the duality between perfect beamforming at the transmitter and maximal ratio combining at the receiver, we will derive the outage probabilities for multiuser MISO systems with perfect beamforming and RVQ beamforming.

A. MU-MIMO TAS Scenario

The outage probability for transmit antenna selection with multiuser diversity in the case of no-delay feedback is given in [9]. We extend the analysis for the case of delayed feedback. During each coherent time, the scheduler selects the i^{th} transmit antenna and the k^{th} user by

$$\{\hat{i}, \hat{k}\} = \underset{i,k}{\operatorname{argmax}} \|\mathbf{h}_i^{(k)}\|^2 \tag{16}$$

where $\mathbf{h}_i^{(k)} = \left(h_{i,1}^{(k)}, \cdots, h_{i,N_r}^{(k)}\right)$ represents the channel vector between the i^{th} transmit antenna and the k^{th} user. Consider that it employs maximal ratio combining (MRC) at the $\hat{k}^{th}$ user, the received signal $\mathbf{r}^{(\hat{k})}$ is given by

$$\mathbf{r}^{(\hat{k})} = \|\mathbf{h}_{\hat{i}}^{(\hat{k})}\|^2 x + \sum_{j=1}^{N_r} \langle \mathbf{h}_{\hat{i},j}^{(\hat{k})}, \mathbf{n}_{\hat{i},j}^{(\hat{k})} \rangle \tag{17}$$

where $\sum_{j=1}^{N_r} \langle \mathbf{h}_{\hat{i},j}^{(\hat{k})}, \mathbf{n}_{\hat{i},j}^{(\hat{k})} \rangle$ is the combined noise with zero mean and variance N_r. The probability density function of $\eta = \|\mathbf{h}_{\hat{i}}^{(\hat{k})}\|^2$ is given by

$$f_\eta(x) = \frac{Z}{(N_r-1)!} x^{N_r-1} e^{-x} \left(1 - e^{-x} \sum_{n=0}^{Nr-1} \frac{x^n}{n!}\right)^{Z-1} \tag{18}$$

where $Z = N_u N_t$.

The outage probability in the presence of feedback delay is

$$\begin{aligned} P_{\text{out}}(R, \epsilon, \rho) &= \Pr\left[\|\tilde{\mathbf{h}}_{\hat{i}}^{(\hat{k})}\|^2 < \frac{2^R - 1}{\epsilon / N_t}\right] \\ &= \Pr\left[\sum_{j=1}^{N_r} \left| \sqrt{2\mu |\mathbf{h}_{\hat{i},j}^{(\hat{k})}|^2} + \sqrt{2} z_j \right|^2 < \frac{2\gamma_0}{1-\rho^2}\right]. \end{aligned} \tag{19}$$

where $\tilde{\mathbf{h}}_{\hat{i}}^{(\hat{k})}$ is defined in (2) and $z_j = \mathbf{e}_{\hat{i},j}^{(\hat{k})} e^{-i\angle \mathbf{h}_{\hat{i},j}^{(\hat{k})}}$ is zero mean complex Gaussian with unit variance. Note that $\sum_{j=1}^{N_r} \left| \sqrt{2\mu |\mathbf{h}_{\hat{i},j}^{(\hat{k})}|^2} + \sqrt{2} z_j \right|^2$ is non-central chi-square distributed with $2N_r$ degrees of freedom and parameter $\sqrt{2\mu\eta}$.

Now, we derive the outage probability for the multiuser MIMO systems with transmit antenna selection and MRC at the receiver.

Proposition 2 (MU-MIMO TAS outage probability)**.** *Consider a N_u-user wireless communication with the base station employing N_t transmit antennas and each user employing N_r receiver antennas employing transmit antenna selection and maximal ratio combining with the transmission date rate of R bits/s/Hz and $\mathsf{SNR} = \epsilon$, the outage probability in the presence of feedback delay is given by*

$$\begin{aligned} &P_{out}^{MUTAS}(R, \epsilon, \rho) \\ &= \frac{N_u N_t}{(N_r-1)!} \sum_{k=0}^{N_u N_t - 1} \binom{N_u N_t - 1}{k} (-1)^k \sum_{m=0}^{k(N_r-1)} \frac{m!\, a_m(N_r,k)}{(1+k+\mu)^m} \\ &\sum_{n=0}^{m} \frac{\mu^n (N_r+n-1)!}{n!(1+k)^{N_r+n}} \binom{N_r+m-1}{N_r+n-1} \Gamma_{N_r+n}\left(\frac{1+k}{1+k+\mu}\beta\right) \end{aligned} \tag{20}$$

where $\beta = \frac{\gamma_0}{1-\rho^2}$ and $a_m(N_r,k)$ is defined in

$$\left(\sum_{l=0}^{N_r-1} \frac{x^l}{l!}\right)^k = \sum_{m=0}^{k(N_r-1)} a_m(N_r,k) x^m. \tag{21}$$

Proof: The proof is provided in [8].□

The diversity order of multiuser MIMO systems with transmit antenna selection and receive maximal ratio combining can be derived as

$$D^{\text{MUTAS}} = \begin{cases} N_r, & 0 \le \rho < 1, \\ N_u N_t N_r, & \rho = 1. \end{cases} \tag{22}$$

Remark: Proposition 1 is a special case of proposition 2 by setting $N_u = N_r = 1$.

It is shown in Fig. 5 that feedback delay has a great impact on transmit antenna selection in multiuser systems, where we show a 2-user wireless communication system with four transmit antennas at the base station and two receive antennas for each user.

B. MU-MISO PBF and RVQ Scenarios

In a N_u-user system with the base station employing N_t transmit antennas and each user equipped with single receive

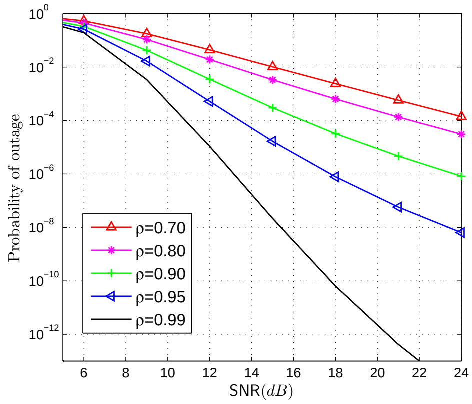

Fig. 5. Outage Probability for 2-user systems with 4 transmit antennas at the base station and 2 receive antennas at each user; the transmission rate is 2 bits/s/Hz.

antenna, the base station select the $\hat{k}^{th}$ user via the following algorithm:

$$\hat{k} = \underset{k}{\operatorname{argmax}} \|\mathbf{h}^{(k)}\|^2. \quad (23)$$

Then $\tau = \|\mathbf{h}^{(\hat{k})}\|^2$ has the following cumulative density function

$$F_\eta(x) = \left(1 - e^{-x}\sum_{n=0}^{N_t-1}\frac{x^n}{n!}\right)^{N_u}. \quad (24)$$

Noticing the duality between perfect beamforming for a $M \times 1$ system and maximal ratio combining for a $1 \times M$ system. we can obtain the outage probability for multiuser MISO systems with perfect beamforming by switching N_r and N_t in equation (20):

$$P_{\text{out}}^{\text{MUPBF}}(R, \epsilon, \rho) = \frac{N_u}{(N_t-1)!}\sum_{k=0}^{N_u-1}(-1)^k \begin{pmatrix} N_u - 1 \\ k \end{pmatrix} \sum_{m=0}^{k(N_t-1)} \frac{m!\, a_m(N_t,k)}{(1+k+\mu)^m} \sum_{n=0}^{m}\frac{\mu^n (N_t+n-1)!}{n!(1+k)^{N_t+n}} \begin{pmatrix} N_t+m-1 \\ N_t+n-1 \end{pmatrix} \Gamma_{N_t+n}\left(\frac{(1+k)\beta}{1+k+\mu}\right). \quad (25)$$

The outage probability for multiuser MISO systems with RVQ beamforming can be easily derived from this equation as

$$P_{\text{out}}^{\text{MURVQ}}(R, \epsilon, \rho) = \frac{N_u}{(N_t-1)!}\sum_{k=0}^{N_u-1}(-1)^k \begin{pmatrix} N_u - 1 \\ k \end{pmatrix} \sum_{m=0}^{k(N_t-1)} m!\, a_m(N_t,k) \sum_{n=0}^{m}\frac{\mu^n (N_t+n-1)!}{n!(1+k)^{N_t+n}} \begin{pmatrix} N_t+m-1 \\ N_t+n-1 \end{pmatrix} \int_0^1 \frac{\nu^n \Gamma_{N_t+n}\left(\frac{(1+k)\beta}{1+k+\nu\mu}\right)}{(1+k+\nu\mu)^m} f_\nu d\nu \quad (26)$$

where f_ν is defined in equation (7).

The diversity order for multiuser MISO systems with perfect beamforming and RVQ beamforming can be given by

$$D^{\text{MUBF}} = \begin{cases} N_t, & 0 \le \rho < 1, \\ N_u N_t, & \rho = 1. \end{cases} \quad (27)$$

V. Conclusion

In this paper we developed a framework to compare the sensitivity of different beamforming techniques, namely Random Vector Quantization and Transmit Antenna Selection, for wireless networks when exposed to delay in the feedback process. The derivation of outage probabilities allowed us to argue on the observable performance degradation in single-user MISO systems and multi-user MIMO systemss. Closed-term expressions for the diversity orders supported our claim. Additionally, we were able to prove through simulation that the increase of codebook size can mitigate the effects of delay.

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